Vehicle Kinematics and Path Following

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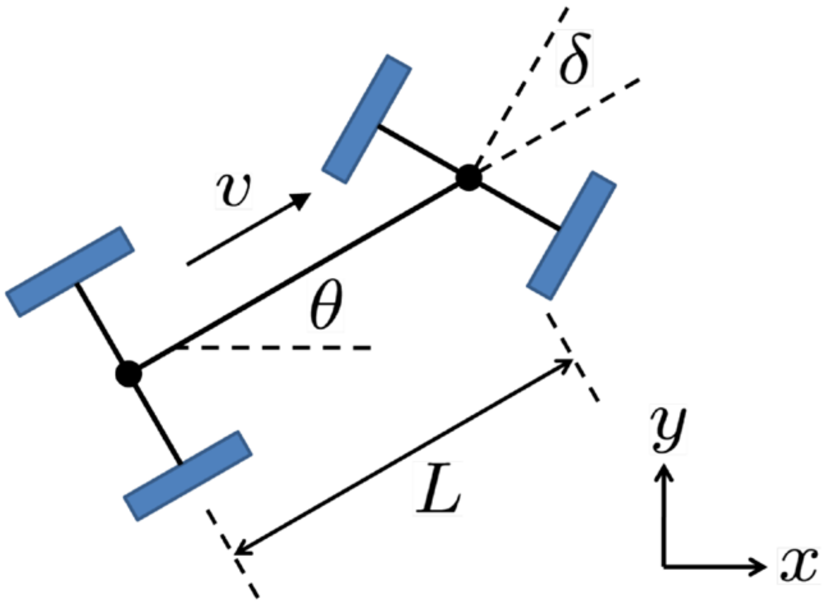
Kinematic Mode: Ackerman Steering

$$\begin{aligned} \dot{x} &= v\cos\theta, \\ \dot{y} &= v\sin\theta, \\ \dot{\theta} &= \frac{v}{L}\tan\delta. \end{aligned}$$

$$\begin{aligned} \dot{\delta} &= \frac{1}{T_\delta}(\delta_R - \delta), \\ \dot{v} &= a, \\ \dot{a} &= \frac{1}{T_a}(a_R - a), \end{aligned}$$

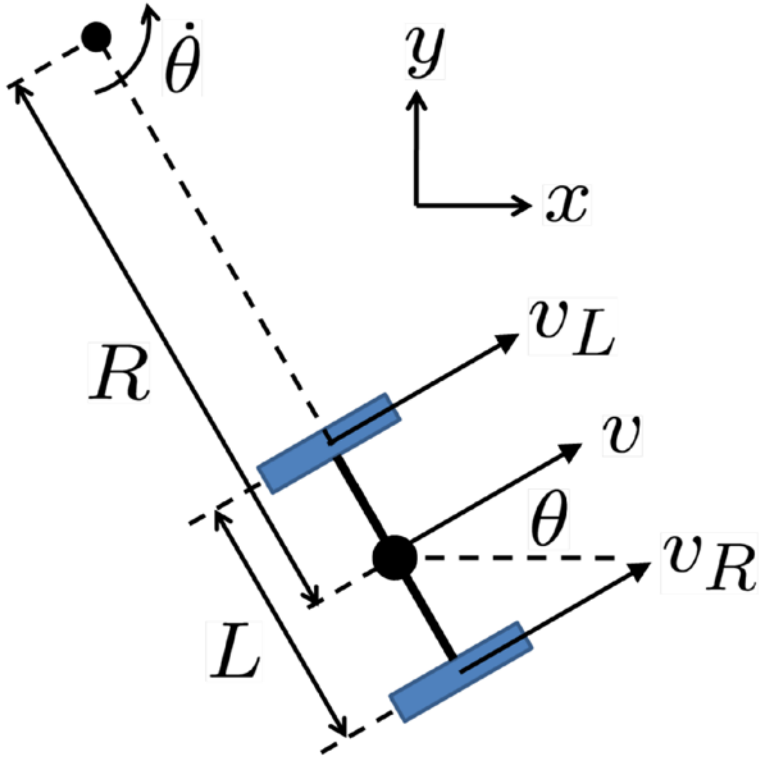


Explicit steering vehicle model

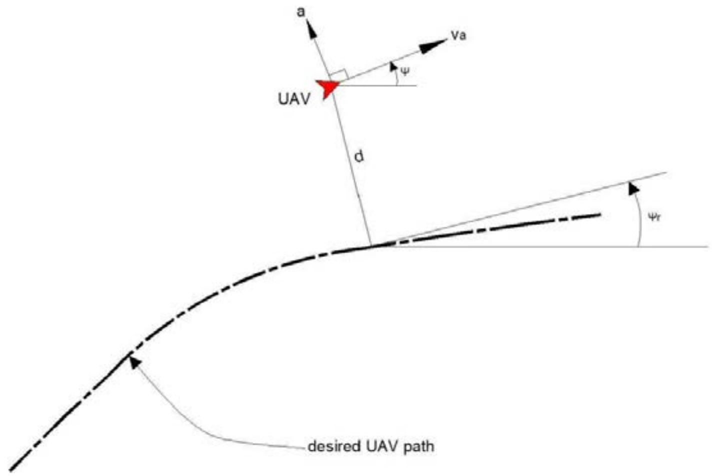
Skid Steering model

$$
\begin{aligned}
\dot{x} &= v\cos\theta, \\
\dot{y} &= v\sin\theta, \\
\dot{\theta} &= \frac{\Delta v}{L}.
\end{aligned}
$$

$$
\begin{aligned}
\dot{x} &= \frac{1}{2}v_R\cos\theta + \frac{1}{2}v_L\cos\theta, \\
\dot{y} &= \frac{1}{2}v_R\sin\theta + \frac{1}{2}v_L\sin\theta, \\
\dot{\theta} &= \frac{1}{L}v_R - \frac{1}{L}v_L.
\end{aligned}
$$



Path Following Problem



- Calculate lateral acceleration

$$\text{(i)} \lim_{t \to \infty} d = 0,$$

$$\text{(ii)} \lim_{t \to \infty} \tilde{\psi} = 0,$$

$$\text{(iii)} |u| \leq u_{max}.$$

Trajectory Generation: Path following

Fixed-wing UAV kinematic model

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} v_a \cos\psi \\ v_a \sin\psi \\ \frac{g}{v_a}\tan\phi \\ -k(\phi - \phi^d) \end{bmatrix}$$

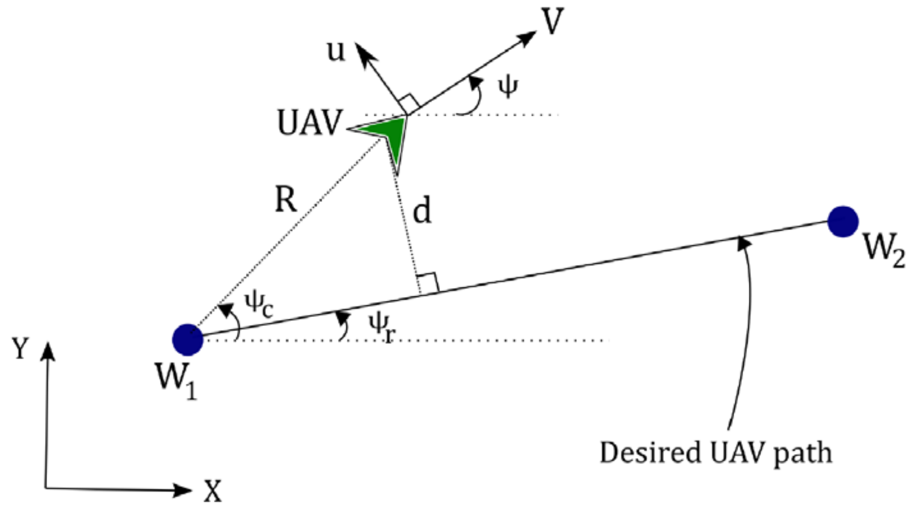
Path following law: pursuit and LOS components

$$\phi^d = \tan^{-1}\left(\frac{k_1(\psi_r - \psi) + k_2 d}{g}\right)$$

Error dynamics

$$\begin{aligned} \dot{d} &= v_a \sin(\psi_r - \psi) \\ \dot{\tilde{\psi}} &= -\frac{k_1}{v_a}(\psi_r - \psi) - \frac{k_2}{v_a} d \end{aligned}$$

Straight Line following



$$
\begin{aligned}
d &= R\sin(\psi_c - \psi_r) \\
\tilde{\psi} &= \psi - \psi_r
\end{aligned}
$$

Circle following

$$d = \sqrt{(x - x_c)^2 + (y - y_c)^2} - R_c$$

$$\tilde{\psi} = \psi - \psi_r$$

$$\psi_r = \tan^{-1}\frac{y_c - y}{x_c - x} \pm \frac{\pi}{2}$$